

## Combined LARES-LAGEOS Solutions

**Krzysztof Sośnica (1), Christian Baumann (1),  
Daniela Thaller (2), Adrian Jäggi (1), Rolf Dach (1)**

(1) Astronomical Institute, University of Bern, Bern, Switzerland.

(2) Bundesamt für Kartographie und Geodäsie, Frankfurt am Main, Germany.  
sosnica@aiub.unibe.ch

**Abstract.** *LARES is a new spherical geodetic satellite designed for SLR observations. It is made of solid tungsten alloy covered with 92 corner cubes. Due to a very small area-to-mass ratio, the sensitivity of LARES orbits to non-gravitational forces is greatly minimized.*

*We processed 82 weeks (Feb12-Aug13) of LARES observations from a global SLR network and we analyzed the contribution of LARES data to the current SLR products (e.g., global scale and geocenter coordinates). The quality of the combined LARES+LAGEOS-1/2 solutions is also addressed in the paper.*

### Introduction

LARES (LAsER Relativity Satellite) was designed by the Scuola di Ingegneria Aerospaziale at the University of Rome and manufactured by the Italian Space Agency (ASI, Ciufolini et al., 2012). The satellite was launched by the European Space Agency (ESA) on February 13, 2012 with the maiden flight of the new ESA small launcher VEGA. LARES was placed in a circular orbit at a height of 1450 km with the inclination  $69.5^\circ$  (see Table 1).

This fully passive spherical satellite is made of a high-density solid tungsten alloy and equipped with 92 fused silica corner cube reflectors (see Figure 1). As opposed to the other geodetic satellites, LARES consists only of one metal layer without a specified inner core (Pavlis et al., 2012). The mass of the satellite is 386.8 kg and the satellite radius is only 18 cm. Therefore, LARES has nowadays the smallest area-to-mass ratio within all artificial satellites ( $2.9 \cdot 10^{-4} \text{ m}^2/\text{kg}$ , i.e., 2.5 times smaller than LAGEOS).

The basic purpose of the satellite mission is to achieve important measurements in gravitational physics, space geodesy, and geodynamics: in particular, together with the LAGEOS-1 and LAGEOS-2 satellites and with the GRACE models, it will improve the accuracy of the determination of Earth's gravitomagnetic field, and of the Lense-Thirring effect. Besides, the satellite can be used for the gravity field determination of low-degree coefficients, estimation of Earth rotation parameters (ERP), and defining the terrestrial reference frame.

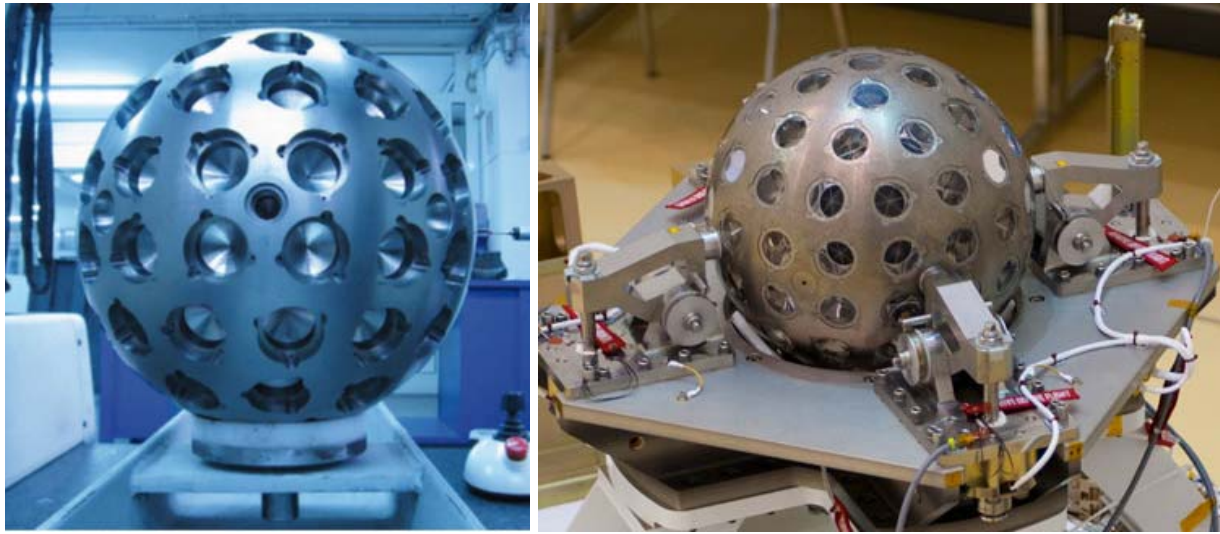
Table 1 shows characteristic periods of LARES' revolution; the drift of ascending node equals 210 days, the drift of perigee equals 376 days, whereas the draconitic year equals 133 days. The draconitic year is a time interval between two consecutive passes of the Sun (in the same direction) through the orbital plane. Most of the non-gravitational forces, related to the perturbations due to the direct or indirect (albedo) solar radiation pressure, impose orbit perturbations with a period of a draconitic year.

### LARES orbits

Table 2 shows the perturbing accelerations due to gravitational, non-gravitational forces, as well as the general relativistic perturbations acting on geodetic satellites. Comparing LARES and AJISAI

(two satellites at similar altitudes), the impact of the gravitational accelerations is nearly the same, whereas the impact of the non-gravitational accelerations is about 22 times smaller for LARES than for AJISAI, due to the much lower area-to-mass ratio of the LARES satellite. Thus, LARES orbits are remarkably well suited for the recovery of the Earth's gravity field or for the verification of the Lense-Thirring effect.

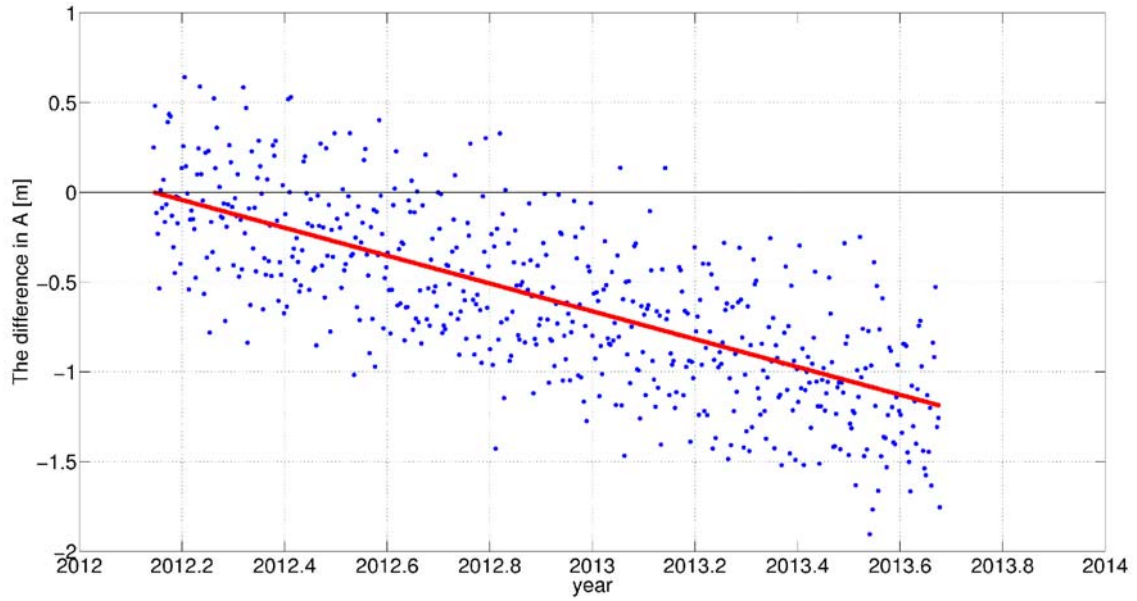
Figure 2 shows the evolution of LARES' mean semi-major axis. The secular drift of the semi-major axis amounts  $-0.775$  m/year, and it is mostly caused by the atmospheric drag, as opposed to the Yarkovsky effect for LAGEOS satellites. LARES' decay of the semi-major axis is about 16 times smaller than AJISAI's decay. The residual LARES' along-track acceleration is just  $-3 \cdot 10^{-12} \text{ ms}^{-2}$ , i.e., 47 times less than the impact of the Lense-Thirring effect.



**Figure 1.** LARES: before embedding the cornercube retro-reflectors (left), and before the launch (right). Courtesy of the Agenzia Spaziale Italiana.

**Table 1.** Characteristics of geodetic SLR satellites. Draconitic years, nodal and perigee's drifts are estimated on the basis of first order orbit perturbations. Secular decays of semi-major axes are estimated on the basis of linear fit to time series of mean semi-major axes  $a$ .

	<b>LAGEOS-1</b>	<b>LAGEOS-2</b>	<b>AJISAI</b>	<b>LARES</b>	<b>Starlette</b>	<b>Stella</b>
Diameter [m]	0.60	0.60	2.15	0.36	0.24	0.24
Mass [kg]	407	405	685	386.8	47	48
Area-to-mass [ $\text{m}^2/\text{kg}$ ]	6.9E-04	7.0E-04	58.0E-04	<b>2.7E-04</b>	9.6E-04	9.4E-04
Radiation coeff. $C_R$	1.13	1.12	1.03	1.07	1.134	1.131
Semi-major axis $a$ [km]	12274	12158	7866	7820	7334	7176
Orbit altitude [km]	5860	5620	1495	1450	812-1113	805
Eccentricity $e$	0.0039	0.0137	0.0016	0.0007	0.0205	0.0010
Inclination $i$ [deg]	109.90	52.67	50.04	69.50	49.84	98.57
Draconitic year [days]	560	222	89	<b>133</b>	73	182
Drift of node $\Omega$ [days]	1050	570	117	210	91	364
Drift of perigee $\omega$ [days]	1694	821	141	376	109	122
Decay of $a$ [m/y]	-0.203	-0.239	-12.000	<b>-0.775</b>	-14.000	-30.000



**Figure 2.** Mean semi-major axis of LARES with a linear fit w.r.t. the initial epoch. The estimated secular decay is  $-0.76 \pm 0.14$  m/year.

**Table 2.** Perturbing accelerations acting on geodetic satellites. Original version: Sośnica (2014)

	<b>LAGEOS-1/2</b>	<b>AJISAI</b>	<b>LARES</b>	<b>Stella</b>
<b>Gravitational perturbations:</b>	[m/s <sup>2</sup> ]	[m/s <sup>2</sup> ]	[m/s <sup>2</sup> ]	[m/s <sup>2</sup> ]
Earth's monopole	2.7	6.4	6.4	7.7
C <sub>2,0</sub>	1.0E-03	6.2E-03	6.3E-03	8.8E-03
C <sub>2,2</sub>	6.0E-06	3.6E-05	3.7E-05	5.1E-05
C <sub>6,6</sub>	8.6E-08	3.1E-06	3.1E-06	6.3E-06
C <sub>20,20</sub>	8.1E-13	1.5E-08	1.6E-08	1.1E-07
Attraction of Moon	2.1E-06	1.9E-06	1.9E-06	1.8E-06
Attraction of Sun	9.6E-07	9.6E-07	9.6E-07	9.6E-07
Attraction of Venus	1.3E-10	1.3E-10	1.3E-10	1.3E-10
<b>General relativity:</b>				
Schwarzschild effect	2.8E-09	1.1E-08	1.1E-08	1.4E-08
Lense-Thirring effect	2.7E-11	1.3E-10	<b>1.4E-10</b>	1.8E-10
Geodetic precession	3.4E-11	4.2E-11	4.2E-11	4.3E-11
<b>Non-gravitational perturbations:</b>				
Solar radiation pressure	3.2E-09	2.5E-08	1.1E-09	4.4E-09
Albedo+infrared radiation	4.4E-10	8.6E-09	3.9E-10	1.8E-09
Thermal reradiation	5.0E-11	4.1E-10	1.9E-11	6.9E-11
Light aberration	1.1E-13	1.1E-12	5.1E-14	2.0E-13
Atmospheric drag (min)	8.0E-15	3.0E-11	2.6E-12	5.0E-11
Atmospheric drag (max)	2.0E-13	5.9E-10	<b>4.8E-11</b>	5.0E-08

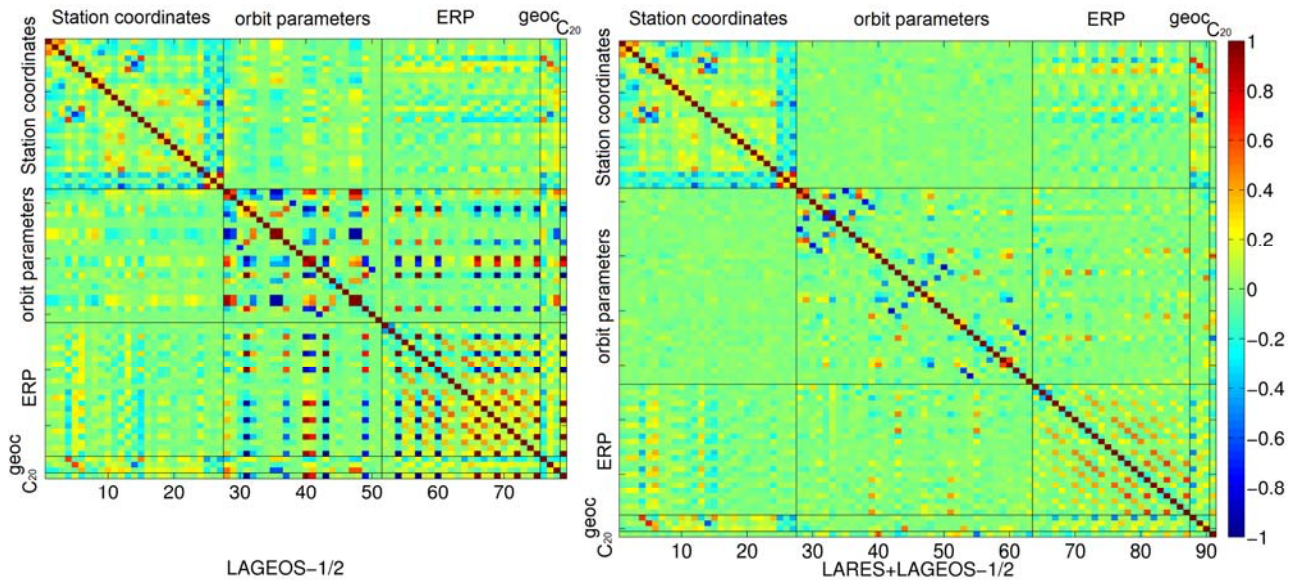
The orbit modeling of LAGEOS satellites follows the description from Sośnica et al., (2013a) Thaller et al., (2013), and Jäggi et al., (2013), whereas the LARES orbits is consistent with modeling of Starlette, Stella, and AJISAI as described in Sośnica et al., (2013b). The range biases are estimated for all stations in case of LARES, because the station-specific center-of-mass corrections are not known. Moreover, the differences of estimated range biases remarkably differ from the a priori values (Baumann et al., 2012, Baumann et al., 2013).

## Correlations

Correlation matrix in Figure 3 (left) shows that the LAGEOS solutions are affected, in particular, by the correlations between:

- orbit parameters & ERP (in particular UT1-UTC),
- station coordinates & orbit parameters,
- geocenter coordinates (the Z component) & orbit parameters,
- ERP & ERP from consecutive days,
- orbit parameters &  $C_{20}$ ,
- $C_{20}$  & UT1-UTC.

Figure 3 (right) shows that by including just one low-orbiting satellite with a different inclination the correlations between estimated parameters are remarkably reduced. In particular the correlations between ERP or orbit parameters and other estimated parameters are diminished. The correlation coefficients between the Z geocenter coordinate and the  $S_C$  orbit parameter (once-per-revolution parameter in along-track), are -0.63, and 0.58 for LAGEOS-1, and LAGEOS-1, respectively in the LAGEOS solutions. These correlations are reduced to -0.24 and 0.21 in the LARES+LAGEOS solutions. The reduction of correlations is similar to that when including Starlette, Stella, and AJISAI to the LAGEOS solutions (Sośnica et al., 2014).



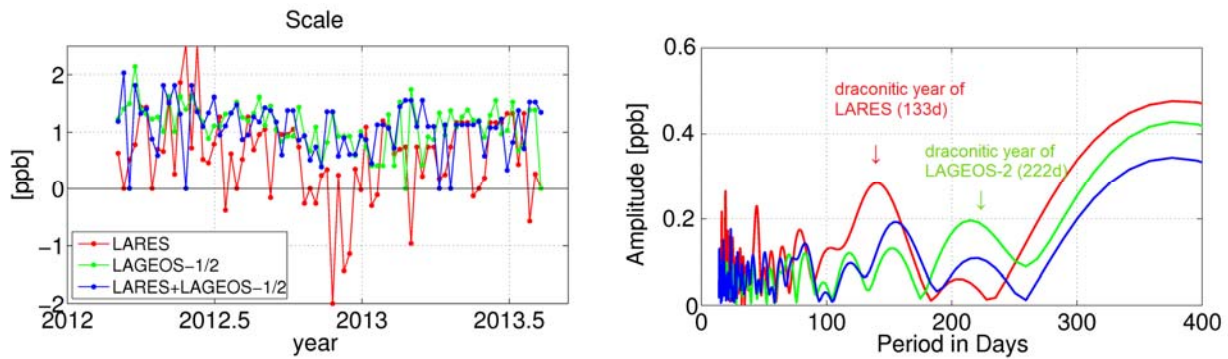
**Figure 3.** Correlation matrices of LAGEOS-1/2 (left) and LARES+LAGEOS-1/2 (right) for a 7-day solution. The matrices contain the core station coordinates, satellites orbits, Earth rotation parameters (ERP), geocenter coordinates, and  $C_{20}$ . All remaining parameters were pre-eliminated (range biases, pseudo-stochastic pulses, non-core station coordinates).



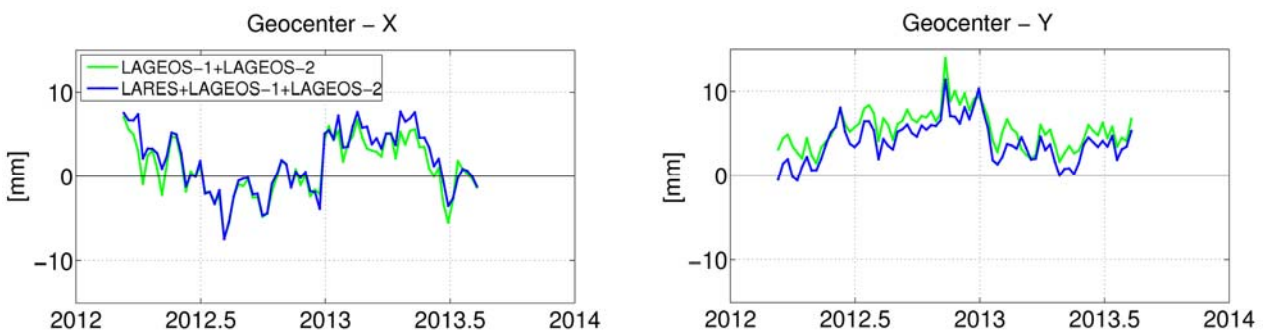
## Reference frame

The scale and origin (geocenter coordinates) of the International Terrestrial Reference Frame (ITRF) are defined by the SLR observations (along with VLBI for the scale). Thus, the highest quality of these parameters is crucial in the SLR solutions. Figure 4 shows the scale from the LARES, LAGEOS, and combined LARES+LAGEOS solutions with a corresponding spectral analysis. The LARES-only scale is noisy and shows some deficiencies in orbit modeling, namely in modeling of the non-gravitational forces which are reflected in the draconitic year of the LARES satellite (e.g., albedo, the Yarkovsky effect, the Yarkovsky-Schach effect, asymmetric satellite reflectivity). The scale defined by LAGEOS-1/2 is stable, but also shows the variations related to the draconitic year of LAGEOS-2. The orbit modeling deficiencies are substantially reduced in the combined LARES+LAGEOS-1/2 solutions, resulting in very stable scale estimates.

Figure 5 shows the geocenter coordinates (x- and y-component as examples). It demonstrates that the inclusion of LARES into LAGEOS solution does not remarkably change an observed signal in geocenter coordinates. Incorporating LARES, however, slightly reduces the offset in the y geocenter component w.r.t. SLRF2008.



**Figure 4.** Scale of reference frame derived from the Helmert 7-parameter transformation of SLR core stations w.r.t. SLRF2008.



**Figure 5.** Geocenter coordinates from the LAGEOS-1/2 solutions and the LARES+LAGEOS-1/2 solutions with respect to the SLRF2008.

## Summary

In the combined LARES+LAGEOS solutions, the correlations between parameters are reduced w.r.t. the LAGEOS-only solutions, and the estimated scale of the reference frame is very stable. A very small impact of non-gravitational forces and a high orbit stability of LARES improve the

quality of the Earth's gravity field recovery, the quality of geocenter and station coordinates, and will allow confirming the special effects of general relativity.

## References

- Baumann C., Sośnica K., Thaller D., Jäggi A., Dach R., Mareyen M., *LARES: Analysis of the first months of data*. Proceedings of the International Technical Laser Workshop 2012 (ITLW-12), Frascati (Rome), Italy, November 5-9, 2012.
- Baumann C., Thaller D., Sośnica K., Jäggi A., Dach R., Mareyen M., *LARES' contribution to GGOS - Assessment after one year in orbit*. EGU General Assembly Vienna, Austria, 2013.
- Ciufolini I., Paolozzi A., Pavlis E., Ries J., Gurzadyan V., Koenig R., Matzner R., Penrose R., Sindoni G., *Testing General Relativity and gravitational physics using the LARES satellite*, Eur Phys J Plus 127:133, 2012.
- Jäggi A., Bock H., Thaller D., Sośnica K., Meyer U., Baumann C., Dach R., *Precise Orbit Determination of Low Earth Satellites at AIUB using GPS and SLR*. ESA Living Planet Symposium, Edinburgh, Scotland, September 09 - 13, 2013.
- Pavlis E., Ciufolini I., Paolozzi A., *LARES: A new ASI mission to improve the measurement of Lense-Thirring effect with satellite laser ranging*. In: Proceedings of the Journées 2011 Systemes de reference spatio-temporels, 19-21 September 2011 - Vienna, 2012.
- Sośnica K., Thaller D., Jäggi A., Dach R., Beutler G., *Sensitivity of Lageos Orbits to Global Gravity Field Models*. Artif Sat 47(2), p. 35-79, 2012.
- Sośnica K., Thaller D., Dach R., Jäggi A., Beutler G., *Impact of loading displacements on SLR-derived parameters and on the consistency between GNSS and SLR results*. J Geod 87(8), p. 751-769, 2013a.
- Sośnica K., Jäggi A., Thaller D., Dach R., Beutler G., *Contribution of Starlette, Stella, and AJISAI to the SLR-derived global reference frame*. Submitted to J Geod, 2013b.
- Sośnica K., Jäggi A., Thaller D., Dach R., Beutler G., Baumann C., *SLR-derived terrestrial reference frame using observations to LAGEOS-1/2, Starlette, Stella, and AJISAI*. In: Proceedings of the 18th International Workshop on Laser Ranging, 11-15 November 2013, Fujiyoshida, Japan, 2014.
- Sośnica K., *Determination of Precise Satellite Orbits and Geodetic Parameters using Satellite Laser Ranging*. PhD thesis of the Philosophisch-naturwissenschaftlichen Fakultät of the University of Bern (in review), 2014.
- Thaller D., Sośnica K., Dach R., Jäggi A., Beutler G., *Lageos-Etalon solutions using the Bernese Software*. In: Proceedings of 17th International Workshop on Laser Ranging, Mitteilungen des BKG, vol 48, pp.333-336, 2012.
- Thaller D., Sośnica K., Mareyen M., Dach R., Jäggi A., Beutler G., *Geodetic parameters estimated from LAGEOS and Etalon data and comparison to GNSS-estimates*. Submitted to J Geod, 2013.